Advanced Automatic Control

"If you have a smart project, you can say "I'm an engineer""

Lecture 2

Staff boarder

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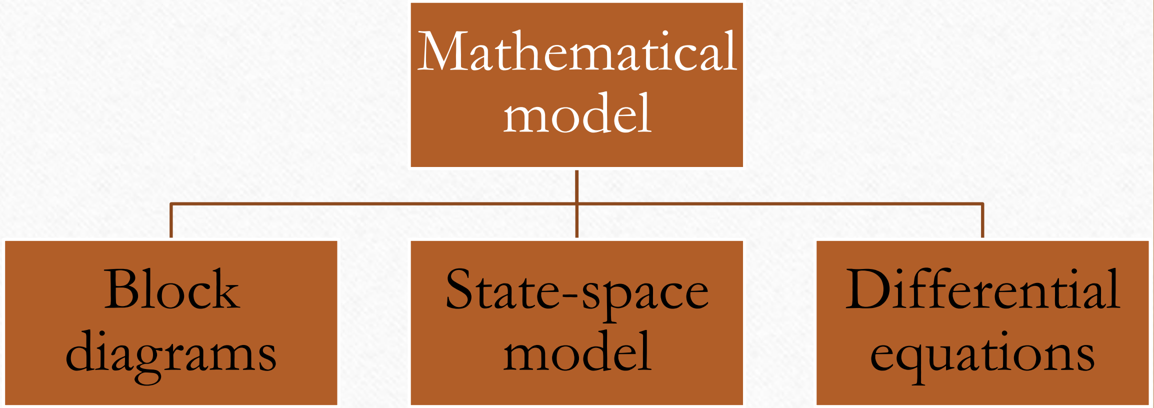
Advanced Automatic Control
MDP 444

- **Lecture aims:**
  - Facilitate combining and manipulating differential equations
  - Identify the equations of motion of systems
  - Understand the mathematical modeling of all systems and combination

Automatic control system

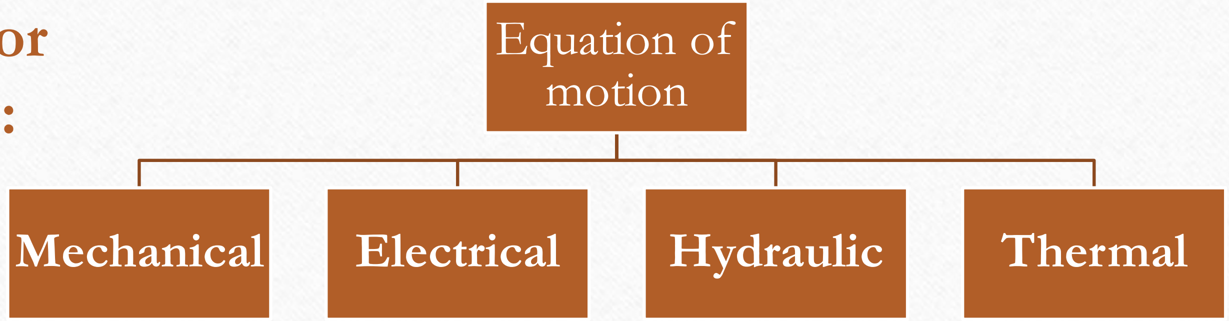
Mathematical Models for the Schematic

- **Understand the physical system and its components**
- **Make appropriate simplifying assumptions**
- **Use basic principles to formulate the mathematical model**
- **Write differential and algebraic equations describing the model**
- **Check the model for validity**



Automatic control system

Must have fundamental method for modelling many physical systems:



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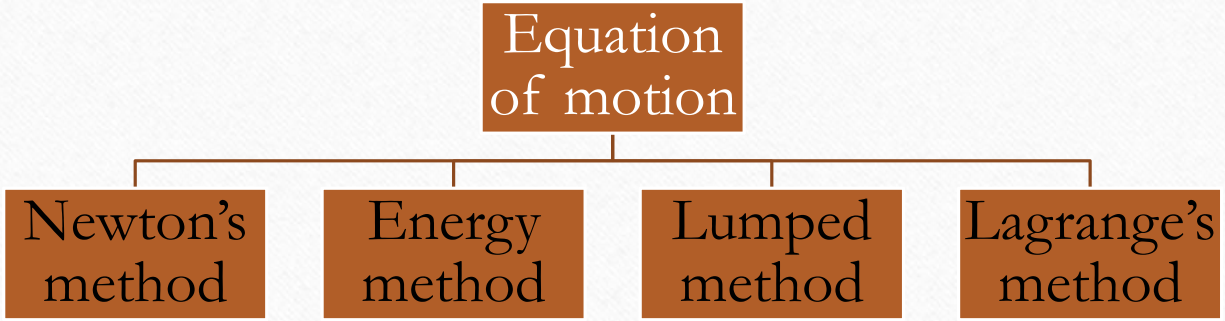
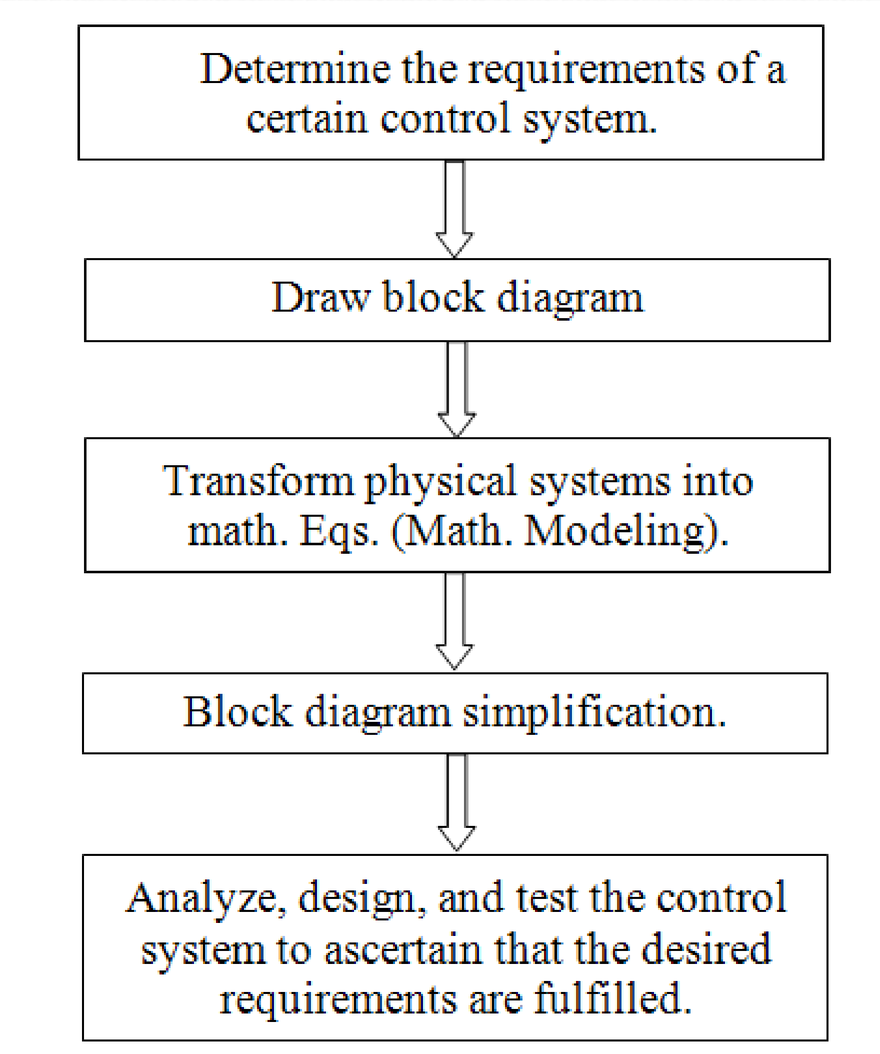


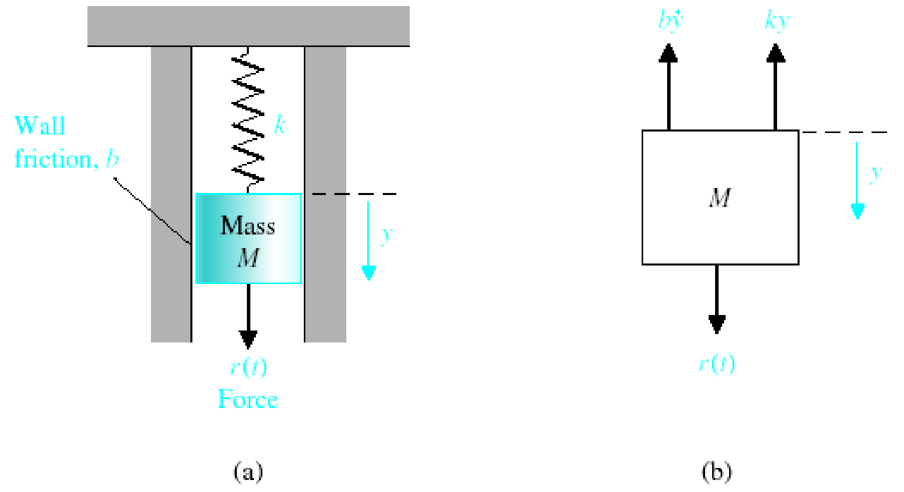
Automatic control system

Procedure



Modeling of Mechanical System

- **Spring - Mass - Damper**

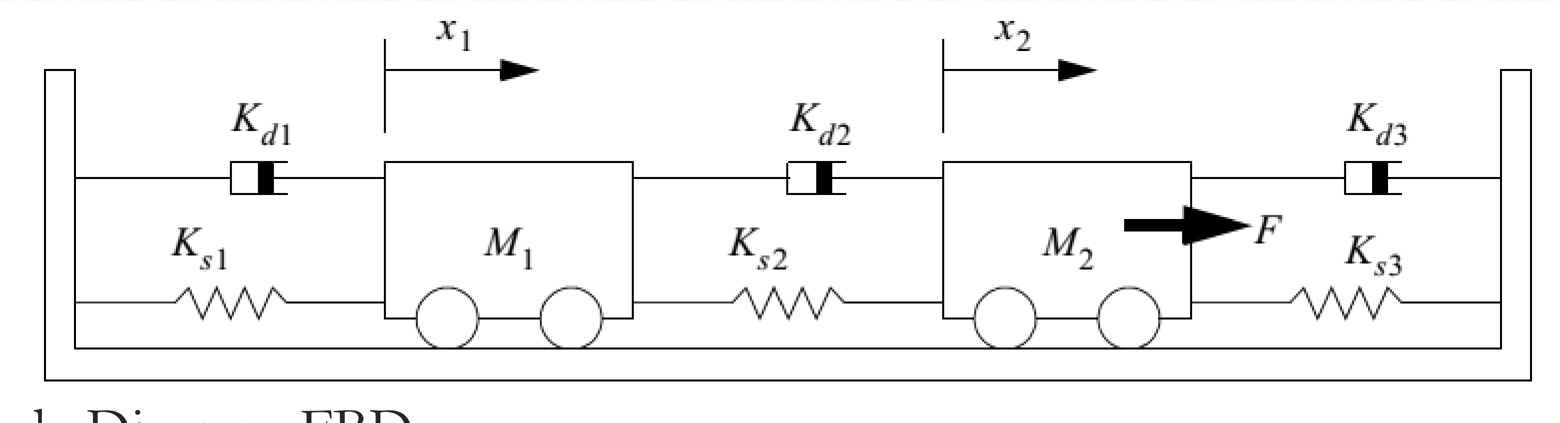


(a) Spring-mass-damper system.
(b) Free-body diagram.

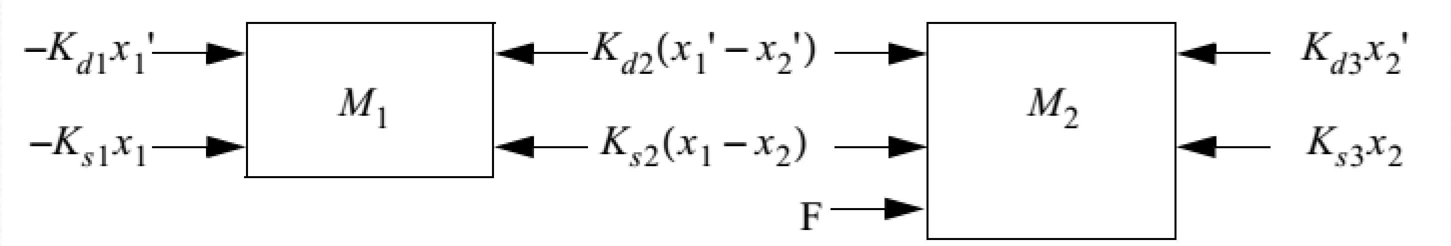
$$M \cdot \frac{d^2}{dt^2} y(t) + b \cdot \frac{d}{dt} y(t) + k \cdot y(t) = r(t)$$

Modeling of Mechanical system

Mathematical Models for the Schematic



- Free Body Diagram FBD

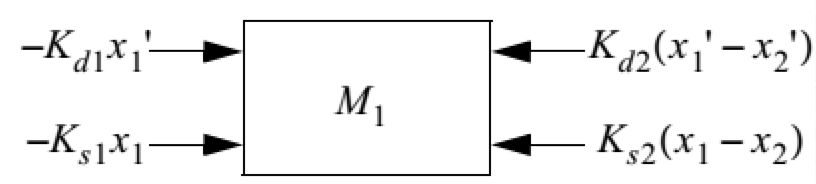


Modeling of Mechanical system

Write equation of motion: Two degree of freedom

Assume X1 >X2 positive direction of motion +→

- For mass(1)

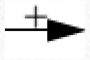


$$\sum F = -K_{d1}x_1' - K_{s1}x_1 - K_{d2}(x_1' - x_2') - K_{s2}(x_1 - x_2) = M_1 x_1''$$

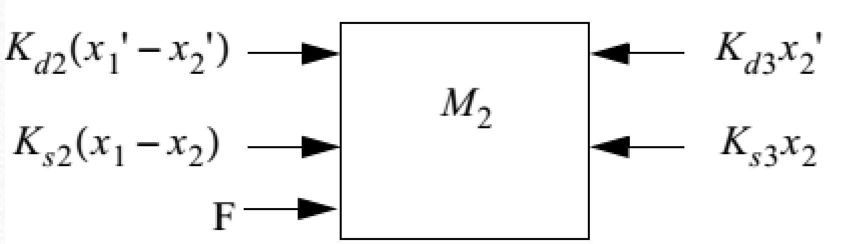
$$x_1''(M_1) + x_1'(K_{d1} + K_{d2}) + x_1(K_{s1} + K_{s2}) + x_2'(-K_{d2}) + x_2(-K_{s2}) = 0$$

Modeling of Mechanical system

Write equation of motion: Two degree of freedom

Assume X1 >X2 positive direction of motion 

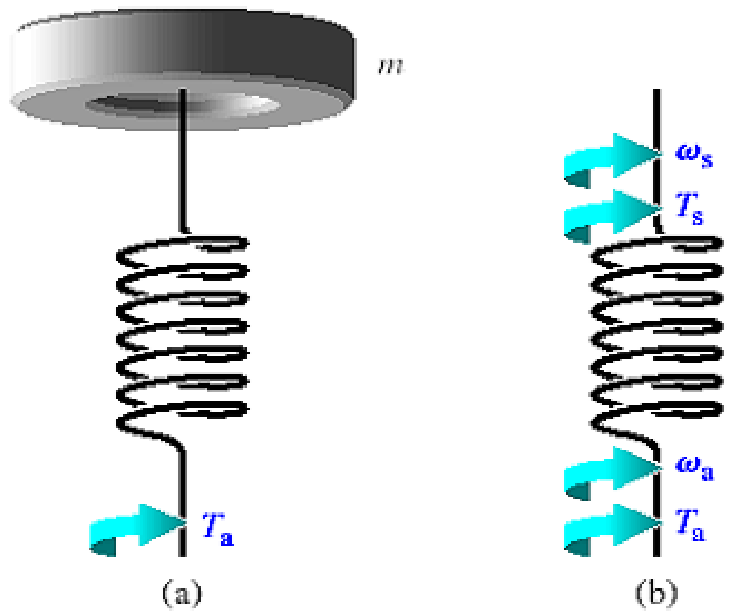
- For mass(2)



$$\sum F = K_{d2}(x_1'-x_2') + K_{s2}(x_1-x_2) + F - K_{d3}x_2' - K_{s3}x_2 = M_2x_2''$$

$$x_2''(M_2) + x_2'(K_{d2}+K_{d3}) + x_2(K_{s2}+K_{s3}) + x_1'(-K_{d2}) + x_1(-K_{s2}) = F$$

Modeling of Mechanical System



(a) Torsional spring-mass system.
(b) Spring element.

$T_a(t) - T_s(t) = 0$

$T_a(t) = T_s(t)$

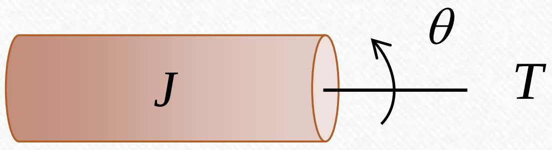
$\omega(t) = \omega_s(t) - \omega_a(t)$

$T_a(t)$ = through - variable

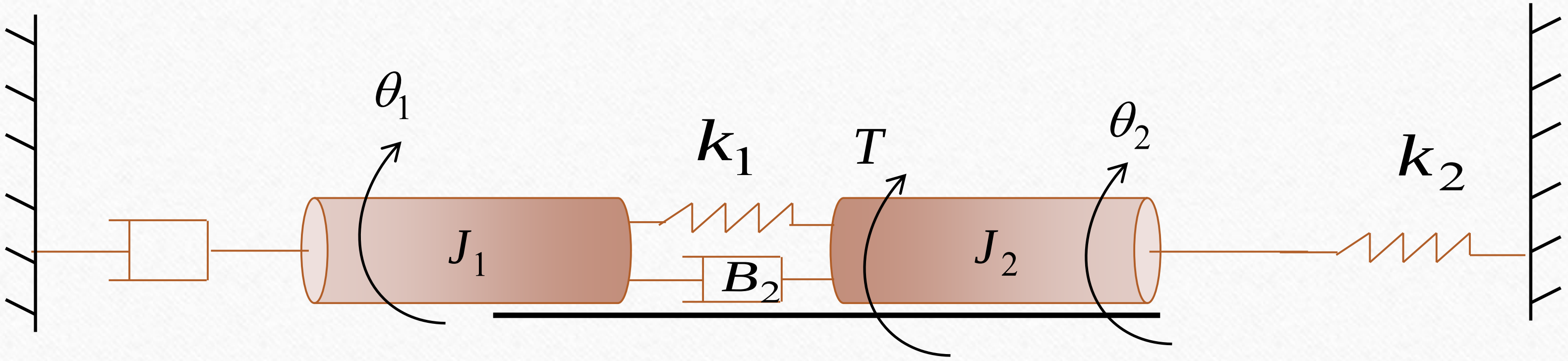
angular rate difference = across-variable

Example

Moment of Inertia



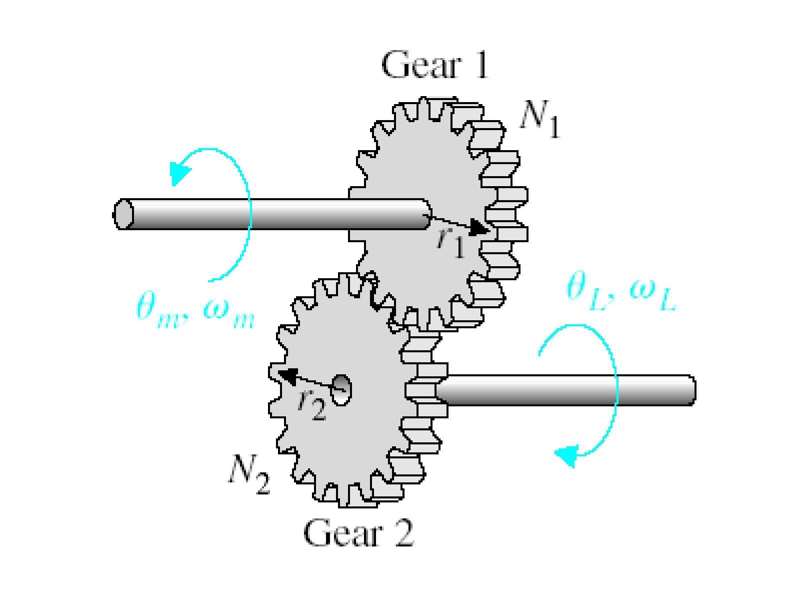
$$T = J\ddot{\theta}$$



Mechanical Building Blocks

Building Block	Equation
	Translational
Spring	$F = kx$
Damper	$F = c\ dx/dt$
Mass	$F = m\ d^2x/dt^2$
	Rotational
Spring	$T = k\theta$
Damper	$T = c\ d\theta/dt$
Moment of inertia	$T = J\ d^2\theta/dt^2$

The Transfer Function of Linear Systems



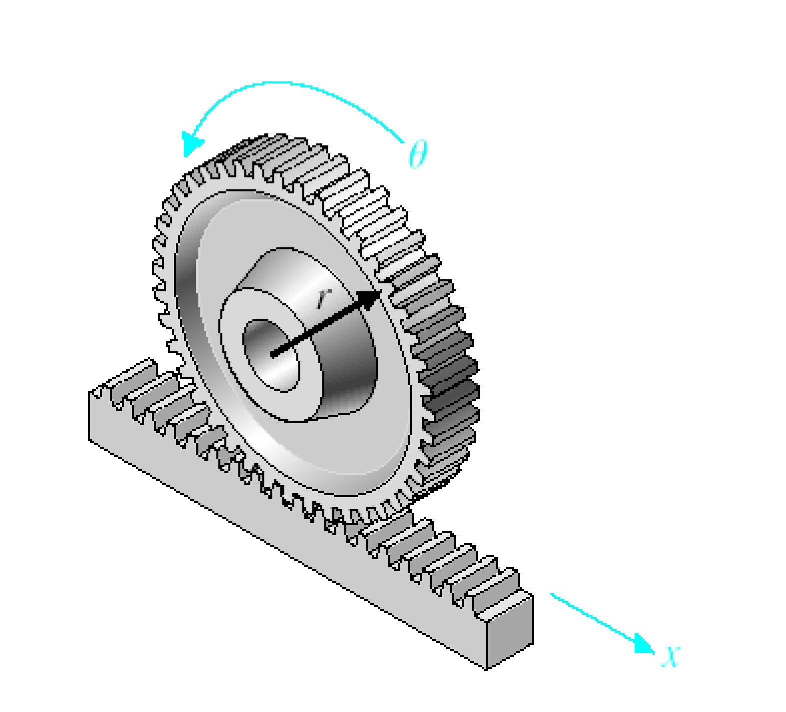
Gear Ratio = n = N1/N2

$N_2 \cdot \theta_L = N_1 \cdot \theta_m$

$\theta_L = n \cdot \theta_m$

$\omega_L = n \cdot \omega_m$

The Transfer Function of Linear Systems



$x = r \cdot \theta$

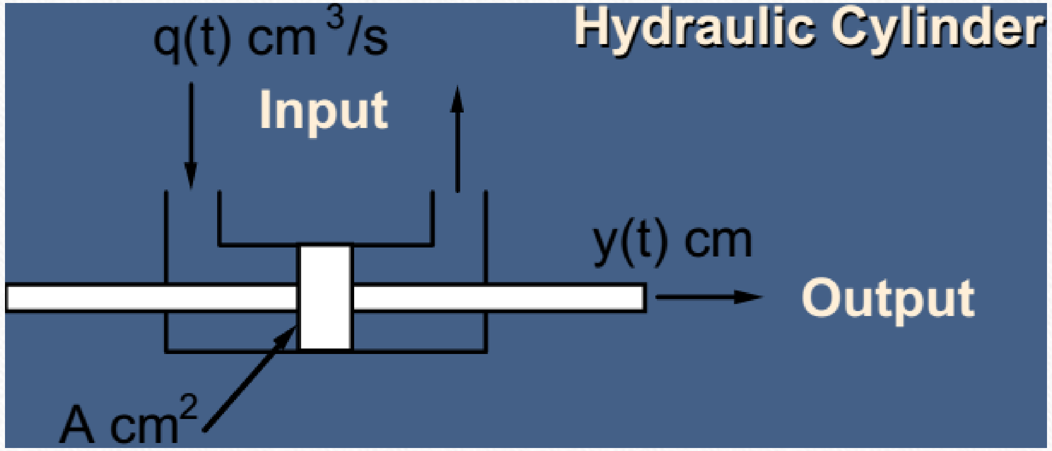
converts radial motion to linear motic

Modeling of Hydraulic System

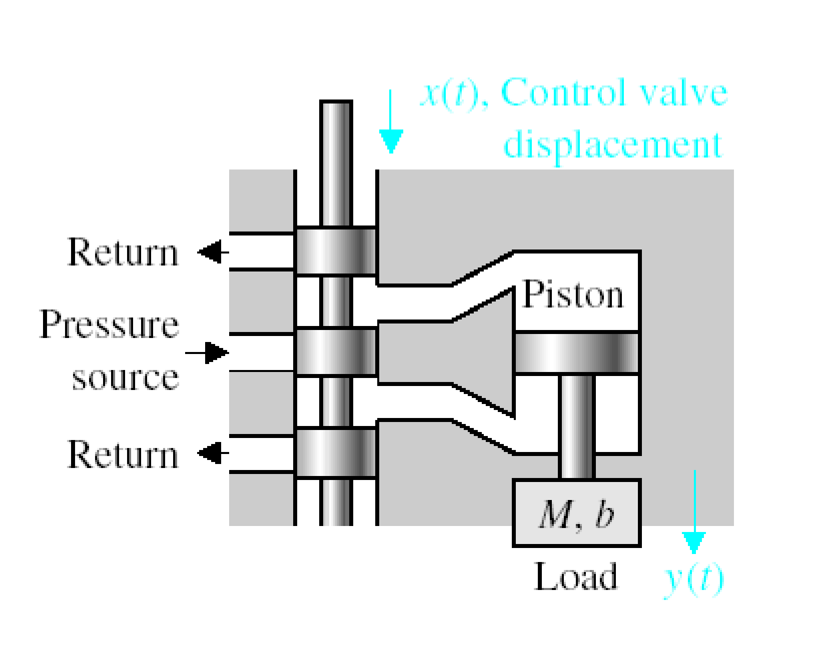
Continuity equation

$$A\,\frac{dy(t)}{dt} = q(t)$$

$$\frac{dy(t)}{dt} = Kq(t)$$



The Transfer Function of Linear Systems



$$\frac{Y(s)}{X(s)} = \frac{K}{s(Ms + B)}$$

$$K = \frac{A \cdot k_x}{k_p} \qquad B = \left(b + \frac{A^2}{k_p}\right)$$

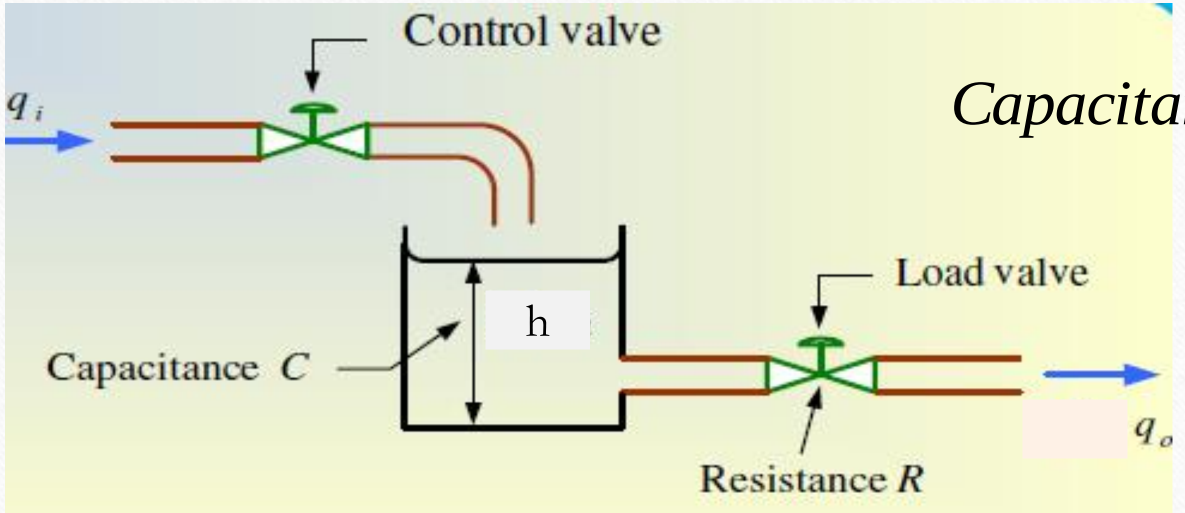
$$k_x = \frac{d}{dx}g \qquad k_p = \frac{d}{dP}g$$

$$g = g(x, P) = \text{flow}$$

A = area of piston

Modeling of Hydraulic System

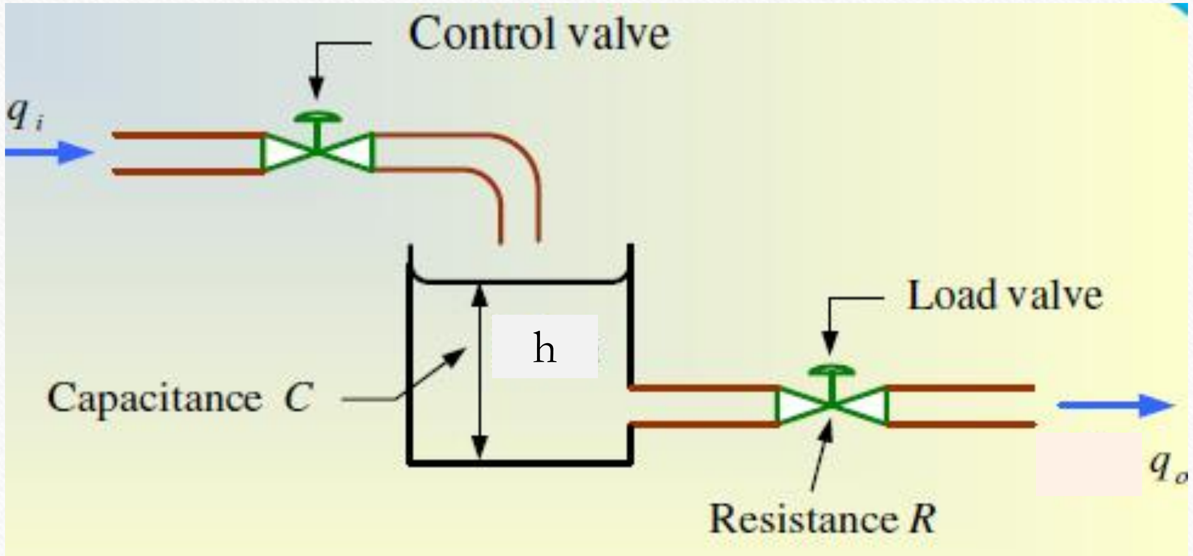
- The capacitance of a tank is defined to be the change in quantity of stored liquid necessary to cause a unity change in the height.



$$Capacitance = \frac{change\ in\ liquid\ stored}{change\ in\ height} = \frac{m^3}{m}\ or\ m^2$$

- Capacitance (C) is cross sectional area (A) of the tank.

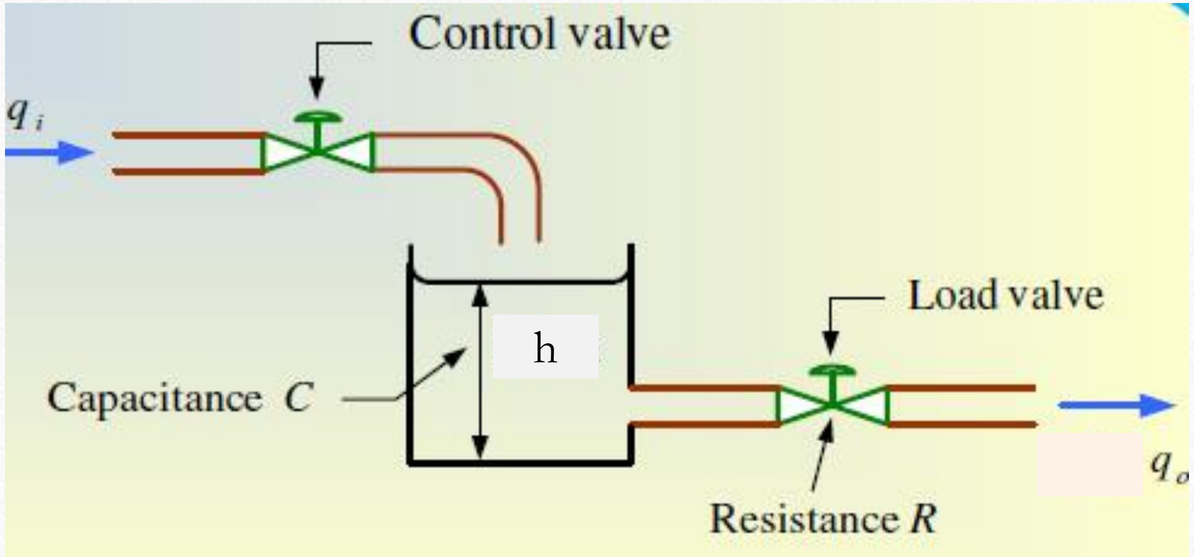
Capacitance of Liquid-Level Systems



$$Rate\ of\ change\ of\ fluid\ volume\ in\ the\ tank = flow\,in - flow\,out$$

$$\frac{dV}{dt} = q_i - q_o \qquad \frac{d(A \times h)}{dt} = q_i - q_o$$

Capacitance of Liquid-Level Systems



$$A\frac{dh}{dt} = q_i - q_o$$

$$C\frac{dh}{dt} = q_i - q_o$$

Modeling of Hydraulic System

Hydraulic Tank

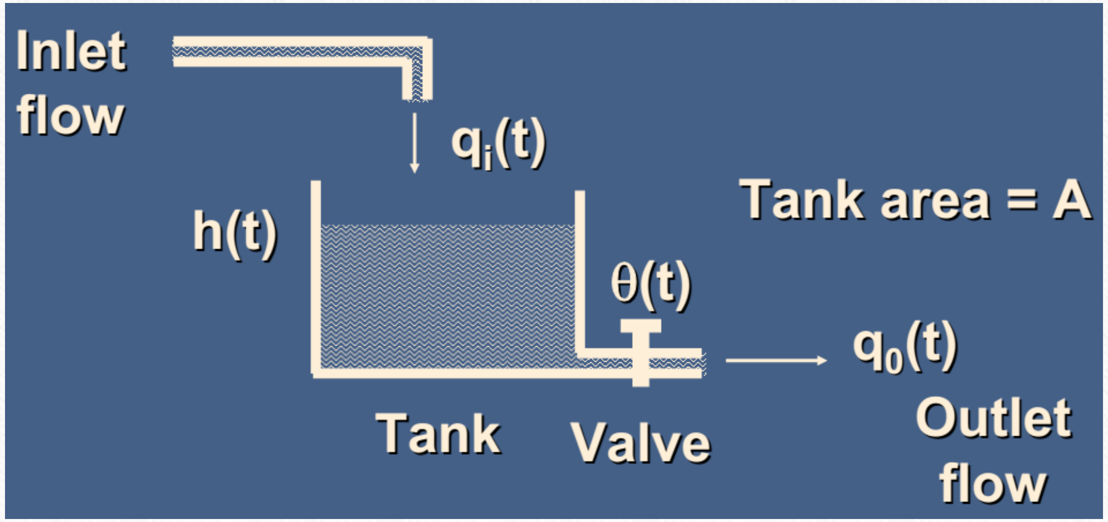
Input

$$q_i(t) - q_o(t) = q_{stored}$$

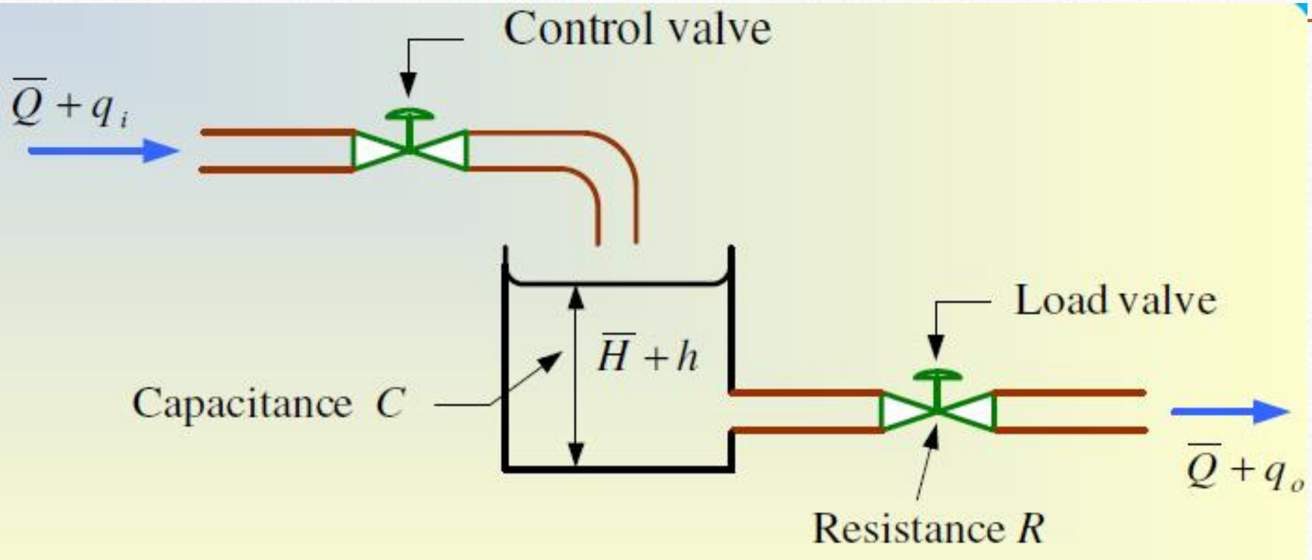
$$q_{stored} = A\,\frac{dh(t)}{dt}$$

$$q_i(t) - q_o(t) = A\,\frac{dh(t)}{dt}$$

$$q_o = \frac{h}{r}$$



Modelling Example



$\overline{H}$ = steady-state head (before any change has occurred), m.
h = small deviation of head from its steady-state value, m.
$\overline{Q}$ = steady-state flow rate (before any change has occurred), m^3/s.
q_i = small deviation of inflow rate from its steady-state value, m^3/s.
q_o = small deviation of outflow rate from its steady-state value, m^3/s.

Modelling Example

- The rate of change in liquid stored in the tank is equal to the flow in minus flow out.

$$C\frac{dh}{dt} = q_i - q_o \longrightarrow \quad (1)$$

- The resistance R may be written as

$$R = \frac{dH}{dQ} = \frac{h}{q_0} \longrightarrow \quad (2)$$

- Rearranging equation (2)

$$q_0 = \frac{h}{R} \longrightarrow \quad (3)$$

Modelling Example

$$C\frac{dh}{dt} = q_i - q_o \longrightarrow (1)$$

$$q_0 = \frac{h}{R} \longrightarrow (4)$$

- Substitute q_o in equation (3)

$$C\frac{dh}{dt} = q_i - \frac{h}{R}$$

- After simplifying above equation

$$RC\frac{dh}{dt} + h = Rq_i$$

- Taking Laplace transform considering initial conditions to zero

$$RCsH(s) + H(s) = RQ_i(s)$$

Modelling Example

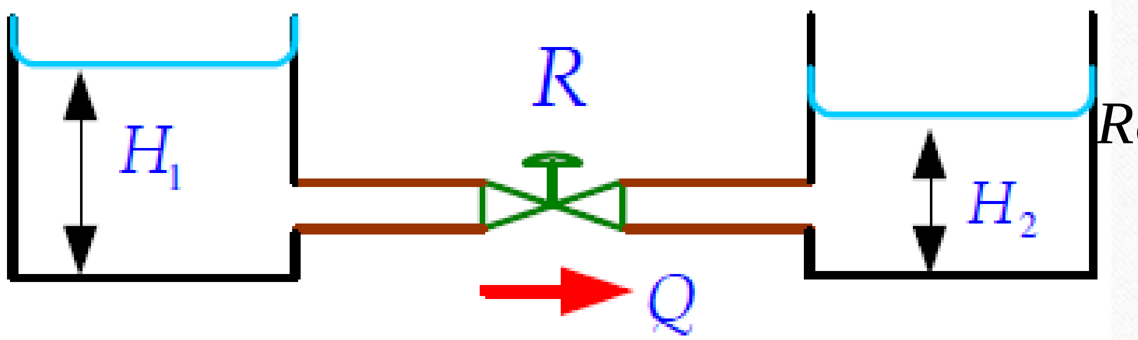
$$RCsH(s) + H(s) = RQ_i(s)$$

- The transfer function can be obtained as

$$\frac{H(s)}{Q_i(s)} = \frac{R}{(RCs+1)}$$

Modeling of Hydraulic System

- The resistance for liquid flow in such a pipe is defined as the change in the level difference necessary to cause a unit change inflow rate.



$$Resistance = \frac{change\ in\ level\ difference}{change\ in\ flow\ rate} = \frac{m}{m^3/s}$$

$$R = \frac{\Delta(H_1 - H_2)}{\Delta Q} = \frac{m}{m^3/s}$$

Model Examples